"On an Instrument for the Measurement of the Length of Long Electric Waves, and also Small Inductances and Capacities."

By J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London. Received January 26,—Read February 9, 1905.

The measurement of the length of the waves used in connection with Hertzian wave telegraphy is an important practical matter. Since the wave-length of the radiated wave is determined by the frequency of the electric oscillations in the radiator, the determination of this frequency is all that is required. The principle of resonance is generally called into assistance to effect this measurement. It may be done by the employment of either an open or a closed resonant circuit.

Instruments for this purpose have been devised in which some circuit having capacity (C) as well as inductance (L) in series with each other, has one or both these factors altered until the electrical time-period of the circuit agrees with that of the wave making circuit. Since this agreement depends upon the equality of the product $\sqrt{CL}$ for the two circuits, the author proposes to call this quantity $\sqrt{CL}$ the *oscillation constant* of the circuit and the number of oscillations in 2π seconds or $2\pi n$, where n is the frequency, the *oscillation number*. Then it is a property of simple oscillatory circuits that the product of the oscillation number and oscillation constant is unity. Some means has, therefore, to be employed to indicate when the adjustment of the two factors of the adjustable circuit has brought its oscillation constant into agreement with that of the transmitter circuit. In a wave-meter devised by J. Dönitz,* which is of the closed circuit form, a condenser of variable capacity has its terminals short-circuited by an inductance coil, and this coil is acted upon inductively by some part of the transmitter circuit so that oscillations are set up in it. A variation of the capacity is made until the root-mean-square value of the current in its circuit is a maximum. This is done by the employment of a sensitive form of hot wire ammeter.

There are, however, some objections to this form of wave-meter, and especially to the use of a hot wire ammeter. The root-mean-square value of the oscillation depends not only upon the maximum value and logarithmic decrement of the oscillations, but upon the number of groups of oscillations per second. Hence, if the discharger of the transmitter is an ordinary spark discharger, the variation of the oscillations due to variation in the break speed or spark-ball surfaces and, therefore of the root-mean-square value of the current set up in

* See J. Dönitz, "On Wave Meters and their Uses," 'Elecktrotechnische Zeitschrift,' vol. 24, p. 920, 1903, Nov. 5; also 'The Electrician,' vol. 52, p. 407, Jan. 1, 1904.

the wave-meter circuit will be considerable. There is, therefore, some difficulty in finding the position of adjustment sharply.

Another point calls for attention. It is well known, from the theory of syntonised circuits, that if two circuits having capacity and inductance are brought into inductive relation to each other, the resulting complex circuit has two time-periods of oscillation. Even if the two circuits when separate and far removed have their time-periods adjusted to equality, the resulting time-period when they are brought into inductive relation to each other differs from the common value. There are, in fact, two frequencies in the coupled circuit, one greater and the other less than the common period. These, however, tend to equality and to indentity with the free independent period of each circuit separately in proportion as the mutual induction between the coupled circuits is reduced.* The object, therefore, held in view in designing the instrument here described was to construct one which, whilst having a fairly large inductance of its own, should be capable of being associated with the circuit to be tested, and set in action by it, by means of a mutual inductance as small as possible.

This has been achieved by making the part of the circuit of the instrument which is acted upon inductively by the circuit to be tested only a small part of the circuit on which its whole inductance depends. We are thus able to keep the self-inductance large and the mutual inductance small, and therefore prevent any great reaction of the secondary current upon the circuit which is being tested.

A closed circuit wave-meter has been devised on these lines by the author, employing a neon vacuum tube detector, which enables the measurement of the oscillation-constant of the transmitter circuit to be made with great accuracy and quickness, and is also useful for measuring small inductances and capacities.†

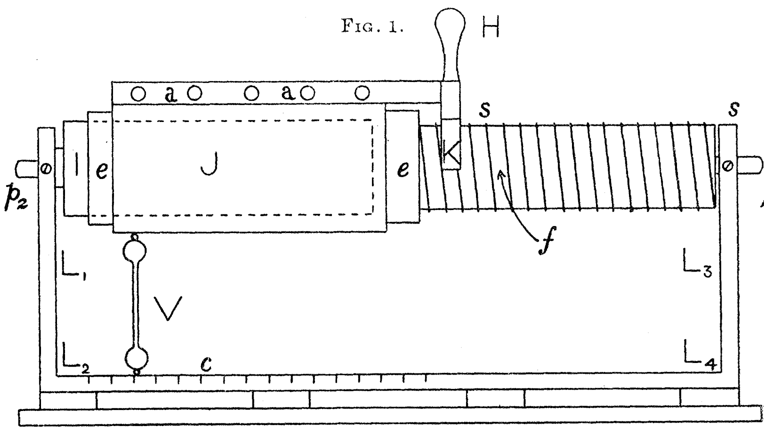
It is constructed in one form, as follows:—An ebonite tube, about 100 cms. long, has cut upon its outer surface a screw groove having 1·6 turns per centimetre and a length of 60 cms. is cut. In this groove is wound a bare copper wire, SS. No. 14 S.W.G. size (see fig. 1), one end of the wire is attached to a metal pin p_1 which forms a bearing supporting the tube, and the other end of the wire is insulated. To this latter end of the ebonite tube is attached a brass tube I 60 cms.

* See A. Oberbeck, "Ueber den Verlauf der Electrischen Schwingungen bei den Tesla'schen Versuchen," 'Wied. Annalen der Physik,' vol. 55, p. 627, 1895.

† The use of a neon vacuum tube as a sensitive indicator of a high-frequency electric field was mentioned first in a paper read by the author to the British Association at Cambridge, September, 1904. The author is indebted to Sir William Ramsay for two tubes of this gas. Sir James Dewar has shown that such neon vacuum tubes can be readily prepared from atmospheric air by the employment of absorbent charcoal at very low temperatures. It would be an advantage if the manufacture of these neon tubes could be placed on a commercial basis, in view of their utility for the purposes here described.

in length and 8 cms. outside diameter. Over this brass tube is placed an ebonite tube *ee* the sides of which are 5 mm. in thickness and its length 80 cms. This ebonite tube fits tightly on the brass tube. The brass tube is closed at the end remote from the inductance coil by a plate and a pin p_2 which forms a bearing for the whole apparatus. Over the larger ebonite tube is slipped a metal cylinder or outer jacket J which can slide easily on the ebonite tube. This jacket carries a rod

FIG. 1.



ending in a half-collar of metal K resting on the inductance spiral (see fig. 2). An ebonite handle H enables the jacket to be moved to and fro. It will be seen, therefore, that the arrangement constitutes a condenser formed of the inner and outer brass tubes separated by an ebonite dielectric, the capacity of which can be varied by moving the outer jacket away from the inner. Also this condenser is in series with an

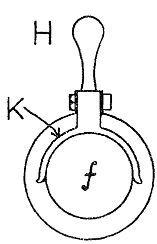


FIG. 2.

inductance coil, the inductance of which is reduced or increased by the same movement which reduces or increases the capacity. The oscillation constant of the arrangement is, therefore, variable between certain limits, and its variation with the displacement of the handle H follows a straight line law.

The pins p_1, p_2 (see fig. 1) which form, respectively, one of the terminations of the inductance and capacity, are connected together by a stout copper bar L_1, L_2, L_3, L_4, so completing the electric circuit, the capacity and inductance of which can be varied. In order to detect the condition in which the oscillations have a maximum value in this circuit, a vacuum tube V is employed (see fig. 1), which may preferably be a vacuum tube containing neon, or failing that, a tube containing rarified carbonic dioxide may be used, the glass being uranium. This vacuum tube should be constructed with two bulbs and with a narrow tubular portion like a spectroscope tube. It may be attached to the outer brass jacket as in fig. 1, or it may be attached to a bar connected with the inner brass tube, the vacuum tube being hung over the outer jacket. Also a scale is provided showing the position of the sliding jacket and which, therefore, can be graduated to show the oscillation constant of the arrangement for various positions of the jacket.

Supposing then that we desire to determine the frequency of the oscillations in any wire such as a Marconi aerial wire used in Hertzian-wave telegraphy, part of this wire is laid alongside the copper bar and the oscillations in it induce others in the circuit of the wave-meter. The oscillation constant is then varied by moving the outer jacket by means of the insulated handle H until the vacuum tube V glows most brightly. If proper adjustments are made of the position of the vacuum tube, it will be found that the tube does not shine at all until the outer jacket J is within a few millimetres of the position in which the oscillation constant of the instrument agrees with that of the circuit being tested. By taking two or three readings with the jacket, a little too far one way and a little too far the other way and approaching the right position from both sides and taking the mean scale reading, it is possible to obtain the oscillation constant with great accuracy. If the capacity is measured in microfarads and the inductance in centimetres, then it will be found that the oscillation constant required is generally some number lying between 1 and 20. The instrument already constructed by the author on this pattern is adapted for the determination of oscillation constants lying between 0·16 and 7·5.

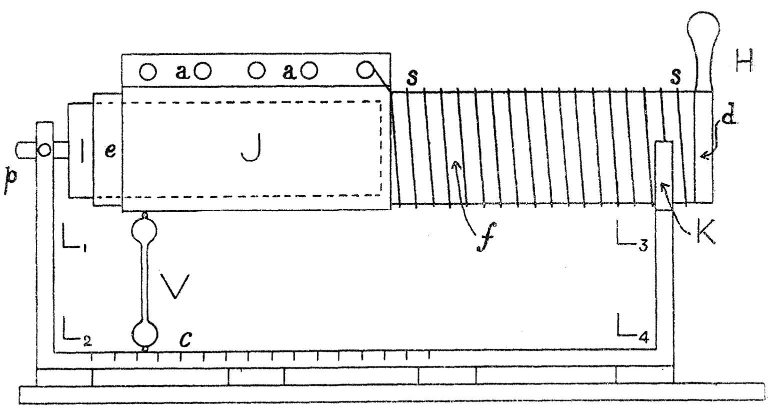
In the case of Hertzian-wave telegraphy conducted on Marconi principles, the wave-length employed, reckoned in feet, is equal to 195·8 times the oscillation constant of the transmitter circuit, or reckoned in metres, to 59·73 times the oscillation constant. Hence, the above described instrument is capable of measuring wave-lengths from 30 to 1500 feet, and might easily be constructed to measure wave-lengths of any greater length.*

By a slight modification, the instrument can be constructed more simply as follows: A single ebonite tube is employed which may be

* *February* 11, 1905.—The author has since constructed one to measure electric waves up to 2400 feet in length.

a metre or a metre and a-half in length and 10 cms. outside diameter, the thickness of the walls being about 5 mm. On this tube is wound as before an inductance coil of 100 or 200 turns of No. 14 S.W.G. bare copper wire. Also the tube is provided with an outer jacket or brass outer tube J (see fig. 3). This is conveniently formed out of the sheet of thin brass bolted round the tube, and one end of the inductance coil SS is attached to this jacket, the other end of the coil being attached to a brass ring *d*, carrying an ebonite handle H. In the interior of this ebonite tube, at one end, there is an inner brass tube I which can slide easily in and out of the ebonite tube *e*. This brass

FIG. 3.

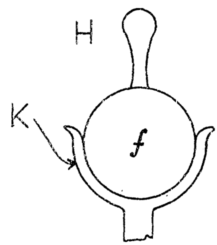


tube has a pin *p* fixed at one end, by which it is attached to a copper bar L_1, L_2, L_3, L_4, ending in a curved crutch or collar K on which the inductance spiral rests (see fig. 4). The other arrangements are as in the instrument already described.

The only differences between the forms shown in fig. 1 and fig. 4 are that in fig. 1 the outer metal jacket is movable and the inner one is fixed, whereas in the second form, the outer jacket is fixed with respect to the inductance coil, and the two together are drawn away from the inner brass tube, thus reducing the capacity and inductance at the same time. The instrument is very convenient for the measurement of small inductances. Thus, for instance, suppose it is desired to measure the inductance of a wire for high frequency currents, the wire having an inductance of not more than a few hundred centimetres, that is to say, something of the order of a microhenry, this inductance may be measured by the instrument in the following manner:—An insulated wire AB (see fig. 5) is laid alongside the copper bar L_2, L_4, and the circuit of the wire is completed by a condenser C of known capacity and a spark gap S. Oscillations are then

set up in this condenser and wire by means of an induction coil as usual, and the handle H is shifted until the vacuum tube V glows or glows most brilliantly. We then know that the oscillation constant of the instrument in that position agrees with that of the circuit so formed. When that is the case, the oscillation constant of the wave-meter can be read off in the scale attached to it, and we, therefore, know the oscillation constant (call it O) of the circuit formed by the condenser and the wire. Hence, if C is the capacity of the condenser in that circuit and L is the inductance of the wire of the circuit, then $O = \sqrt{CL}$. If, then, we increase the inductance L by adding in series with it a wire of which the inductance is desired (call it L′), we can

FIG. 4.



then shift the handle H until we get a fresh agreement and find a second value O′ for the oscillation constant of the circuit. Then we have

$$O' = \sqrt{C(L + L')}.$$

Hence
$$L' = \frac{O'^2 - O^2}{C}.$$

As an illustration the following test measurement was made. A copper wire, the diameter (d) of which was 0·128 or 0·32 cm. was bent into a nearly complete circle 70 cms. in diameter. The inductance of this wire can be calculated from the formula

$$L' = 2\pi D\left(\log_e \frac{4\pi D}{d} - 2{\cdot}45\right) + \frac{R'}{2\pi n}.$$

In the above case $D = 70$ cms., and $d = 0{\cdot}32$ cm., and R' is the resistance of the wire to oscillations having a frequency n. The value of L′ is then 2400 cms. This wire was joined in series with another wire laid alongside the bar L_2, L_4 of the measuring instrument and a condenser having a capacity of 0·00146 microfarad joined up in series with the circuit and a spark gap. The instrument was then used to determine the oscillation constant O′ of the circuit with the circular

wire included, and the oscillation constant O when the circular wire was not included. It was found that $O' = 3{\cdot}5$ and $O = 3{\cdot}0$. Whence

$$L' = \frac{12{\cdot}25 - 9}{0{\cdot}00146} = 2226 \text{ cms.},$$

a value in fairly close agreement with the calculated value, considering that the inductance is less than 2·5 microhenrys.

In this manner it is possible to determine the inductance of a foot or two of coiled copper wire for high frequency currents with fair accuracy. The oscillation constant of an instrument of the above form for various positions of the outer jacket or inner jacket, according to the form used, can best be determined directly by means of a standard wave-meter, such as that described by the Author in a paper to the British Association at Cambridge, September, 1904.*

In this last mentioned arrangement, a long ebonite tube is wound over uniformly with a fine silk-covered copper wire in closely adjacent turns and in one layer. The capacity and inductance per unit of length (c and l) of this long helix must then be determined by known methods, and from this measurement we can determine the velocity of propagation of an electric wave along the helix, for it is equal to the reciprocal of the square root of the product of the capacity and inductance per unit of length of the helix. If we form an oscillatory circuit (see fig. 5), consisting of a condenser, C, a variable inductance, L, and a spark gap, S, the variable inductance including a length of straight wire AB, which can be placed parallel with, and close to, the copper bar of the form of wave-meter described in this paper, then we can bring its oscillation constant into agreement with the oscillation constant of the circuit formed of the variable inductance and condenser. In order to determine the value of this oscillation constant we connect the long helix above described to one terminal of the condenser of the oscillating circuit above described, one spark ball being to earth. The arrangement must be as shown in fig. 5. The long helix of insulated wire HH is then provided with a sliding metal saddle D, which can be connected to the earth E, and this saddle is moved along the helix until a position is found such that by means of a neon or other sensitive vacuum tube V, we can detect a node of potential half-way between the saddle and the point of attachment of the helix to the oscillating circuit, the saddle itself being connected to the earth will also be a node of potential. Hence the distance between the saddle and the end of the helix attached to the oscillating circuit is equal to one wave-length of the wave travelling along the helix.

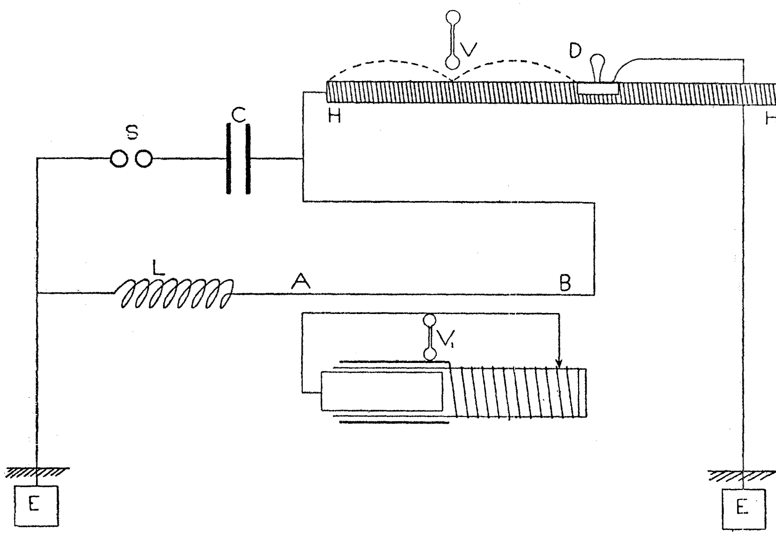
* See J. A. Fleming, "On the Propagation of Electric Waves along Spiral Wires, and on an Appliance for Measuring the Length of Waves used in Wireless Telegraphy," 'Phil. Mag.,' October, 1904.

If W is the velocity of the wave along the helix and λ is the wave-length of the stationary wave, as measured on the helix, and n the frequency of the oscillations, then $W = n\lambda$. If C is the capacity and L the inductance in the oscillatory circuit formed with the condenser of unknown capacity and variable inductance, then the frequency in this circuit is $n = 1/2\pi\sqrt{CL}$.

Also if c and l are the capacity and inductance per unit of length of the long helix, we have $W = 1/\sqrt{cl}$. Therefore

$$\frac{1}{\sqrt{cl}} = \frac{\lambda}{2\pi\sqrt{CL}}, \quad \text{or} \quad \sqrt{CL} = \frac{\lambda}{2\pi}\sqrt{ct}.$$

FIG. 5.



Hence, since λ can be measured, and the oscillation constant of the helix per unit of length $\sqrt{cl}$ is known, we have the oscillation constant of the exciting circuit, and therefore of the closed circuit wave-meter in any position when it is adjusted to agree with that circuit. Otherwise, if we have the means at hand, the oscillation constant of the instrument can be determined for various positions of the slider by simply measuring the capacity (C) of the condenser and inductance (L) of the helix which are effective in that position, and calculating the value of $\sqrt{CL}$ for various positions of the outer or inner jacket, according to the form of instrument used. The instrument can have its scale marked to show directly either oscillation constants (O) or frequencies (n), or aërial wave lengths (λ) in metres or feet. The instrument is not only useful for quantitative work in connection with Hertzian-wave telegraphy, but

is very useful in the laboratory for the determination of the coefficients of coupling and mutual inductances of oscillation transformers.

If there be two circuits inductively connected, forming an air core transformer or oscillation transformer, and if these circuits have respectively coefficients of self-induction L and N, and a coefficient of mutual induction M, then we can determine L and N and M, if these are not too large, by the instrument, and also the "coefficient of coupling" of the transformer, viz., $M/\sqrt{LN}$.

For this purpose we connect the two circuits of the transformer in two ways. 1st, so that a current sent through the circuit flows round in the same direction in the two coils; and 2nd, so that it flows in the opposite direction in the two coils. In the first position, the effective inductance of the whole system is L_1, where $L_1 = L + 2M + N$, and in the second position it is L_2, where $L_2 = L - 2M + N$. Hence, $L_1 + L2 = 2(L + N)$, and $L_1 - L_2, = 4M$. Accordingly, we make four inductance measurements with the instrument. 1st, that of the primary coil alone, the secondary being open; 2nd, that of the secondary alone, the primary being open; 3rd, that of the primary and secondary together joined up to assist; and 4th, that of the primary and secondary together, joined up to oppose.

Twice the sum of the first two measurements should agree with the second, and one quarter of the difference of the last two gives the mutual inductance. Having therefore L, M, and N, we can calculate the coupling, $M/\sqrt{LN}$.

In making measurement of this kind with high frequency currents it is necessary to bear in mind that we cannot obtain the true separate inductance of the primary coil simply by measuring it with the secondary coil over it, even if that secondary coil has its terminals open. There is a quite sensible dielectric current which passes from turn to turn of the secondary coil when over the primary, even if that secondary coil is open, and this dielectric current has the effect, in accordance with well-known principles, of reducing the effective inductance of the primary circuit. Nevertheless, in the above measurement it is proper to take as L the inductance of the primary measured in contiguity to the open secondary, and as the value of N the inductance of the secondary measured in contiguity to the open primary.

As an example of such a measurement the following figures may be given. A certain air-core transformer had a primary consisting of one turn of thick stranded copper wire, and a secondary of eight turns of thinner stranded wire. Measuring as above described, the following values in centimetres were found by the appliance here described:—

$$L = 695 \text{ cms.}, \qquad N = 45{,}700 \text{ cms.}$$

$$L_1 = L + 2M + N = 53{,}000 \text{ cms.}$$

$$L_2 = L - 2M + N = 40{,}120 \quad ,,$$

Hence $$M = \tfrac{1}{4}(L_1 - L_2) = 3{,}220 \text{ cms.},$$

and $$\tfrac{1}{2}(L_1 + L_2) = 46{,}560 \quad ,,$$

whilst from the independent measurements of L and N as above we have,

$$L + N = 46{,}385 \text{ cms.}$$

Hence, the agreement between the last two sums is fairly close. Also the coupling $M/\sqrt{LN}$ is found to be equal to 0·57.

It is usual to call the "coupling" of a primary and secondary coil "close" if it exceeds in value 0·5, and "loose" when it is less than 0·5.

The wave or frequency meter enables us to exhibit in the form of an attractive lecture experiment the well-known fact that the closing of the secondary circuit of an induction coil or transformer reduces the effective inductance of the primary coil. Also since it enables us to determine the frequency (n) of a high frequency current, and it enables us to determine also the value of the high frequency resistance R′ of a round sectioned copper wire of which the diameter d and steady or ordinary resistance R is known, since

$$R' = \frac{\pi d}{80}\sqrt{n} \,.\, R,$$

it becomes, therefore, a useful addition to laboratory appliances.

In Hertzian-wave telegraphy the varying power of waves of various lengths to travel over land or sea surfaces is well known, and it is, therefore, a practical necessity to be able to measure the wave-lengths of the wave sent out. The wave-meter enables us to conduct a kind of spectroscopy on a gigantic scale when we are operating with electric waves hundreds of feet in length instead of fractions of an inch.

We can by means of it discover, for instance, that a wave 300 feet in length travels well over a sea surface, but will not go across a city. On the other hand, the author has been able to communicate well across London by means of electric waves 1000 feet in wave-length.

[*Note added, February* 14, 1905.—The above-described instrument enables us to show that, in the case of an aërial wire or antenna, as used in wireless telegraphy, inductively coupled to a condenser exciting circuit, even if the two circuits, open and closed, have separately the same electrical time period, yet, when coupled, there are two waves radiated of different wave-lengths and frequencies, differing also in period from the free separate time period of each circuit. This result, predicted by theory, is confirmed by experiment.

A name is required by which to designate the instruments here

described, and others of similar nature. The word wave-meter may probably be preferred in practice, but, if a special term is desired, the author suggests, with diffidence, the name *Cymometer* or *Kymometer* (from κῦμα, a wave) as applicable to it.*

"The Effects of Momentary Stresses in Metals." By Bertram Hopkinson, M.A., Professor of Mechanism and Applied Mechanics in the University of Cambridge. Communicated by Professor Ewing, F.R.S. Received January 31,—Read February 16, 1905.

In 1872 the late Dr. John Hopkinson published an investigation into the effect of a blow delivered by a falling weight on the lower and free end of a wire, the upper end of which is fixed.† It is unnecessary to repeat the mathematical analysis in full, but its main features appear in the following argument:—As soon as the weight strikes the stop at the lower end a wave of extension starts up the wire, and the velocity with which it is propagated is $\sqrt{E/\rho} = a$, where E is Young's modulus, and ρ the density of the wire. At a time t after the weight has struck, so short that its velocity is not appreciably diminished, the lower end of the wire has moved through a distance Vt, where V is the velocity of the weight immediately after striking. That is to say, the wire as a whole is lengthened by an amount Vt. This extension is felt over a distance at from the lower end, that being the distance through which the wave of extension initiated by the blow has travelled. The mean strain in this portion of the wire is therefore V/a, and the remainder of the wire is not extended. The wave now travels up the wire to the fixed end, and when it reaches there a reflected wave of equal amplitude starts down the wire. There results momentarily at the top end of the wire a strain equal to $2V/a$ with a corresponding tension $2EV/a$. This is the maximum tension experienced by any part of the wire until the reflected wave again reaches the lower end.

Each bit of the motion of the weight after striking contributes an element to the wave of extension, which is proportional to the then velocity of the weight. The weight is continually being retarded, and the amplitude of the wave therefore continually diminishes as you go back from its front.

* The writer is indebted to his colleague, Professor A. Platt, for advice on the correct form of these words.

† 'Original Papers,' Hopkinson, vol. 2, p. 316.